Theory of Equations

- The theory of equations is a branch of mathematics that deals with the study of equations and their solutions. It encompasses various concepts and techniques to understand and solve equations of different types. The theory of equations has applications in several areas of mathematics and science, including algebra, calculus, physics, engineering, and computer science.

Types of equations

- Linear Equations: Linear equations are equations of the form $ax + b = 0$, where a and b are constants and x is the variable. They have degree 1 and the variable appears only to the power of 1. Examples include $2x + 3 = 7$ and $4y - 5 = 2y + 1$.

- Simple Equations: A mathematical equation which represents the relationship of two expressions on either side of the sign. It mostly has one variable and equal to symbol. Example: $2x – 4 = 2$. In the given example, x is a variable.

- Simultaneous equations, also known as a system of equations, are a set of equations with multiple variables that are to be solved together. The goal is to find values for the variables that satisfy all the equations in the system.

  A general form of a simultaneous equation with two variables, x and y, can be written as:

  Equation 1: $ax + by = c$ Equation 2: $dx + ey = f$

  Here, a, b, c, d, e, and f are constants, and x and y are the variables we are trying to solve for.

- **<u>The substitution method</u>** is a technique used to solve a system of equations algebraically. It involves solving one equation for one variable and substituting that expression into the other equation(s) to eliminate that variable, eventually leading to the solution.
- Here's a step-by-step explanation of how the substitution method works:
- **Consider a system of two equations with two variables:**
  Equation 1: $y = f(x)$ Equation 2: $y = g(x)$
- Solve one of the equations for one variable. Let's say we solve Equation 1 for y: Equation 1: $y = f(x)$ (no change) Equation 2: $y = g(x)$
- Substitute the expression for y from Equation 1 into Equation 2: Equation 2: $f(x) = g(x)$
- Solve the resulting equation for the variable x. This will give you the value(s) of x.
- Substitute the value(s) of x into one of the original equations (either Equation 1 or Equation 2) to find the corresponding value(s) of y.
- The solution to the system of equations is the set of values (x, y) that satisfy both equations.
- **Here's an example to illustrate the substitution method:**
- Example: Consider the system of equations: Equation 1: $2x + y = 5$ Equation 2: $3x - y = 2$
- Step 1: Solve Equation 1 for y: Equation 1: $y = 5 - 2x$
- Step 2: Substitute the expression for y into Equation 2: $3x - (5 - 2x) = 2$
- Step 3: Simplify and solve for x: $3x - 5 + 2x = 2$ $5x - 5 = 2$ $5x = 7$ $x = 7/5$
- Step 4: Substitute the value of x into Equation 1 or Equation 2: Using Equation 1: $y = 5 - 2(7/5) = 5 - 14/5 = 25/5 - 14/5 = 11/5$
- Step 5: The solution to the system of equations is $x = 7/5$ and $y = 11/5$.

- **The elimination method** also known as the method of elimination or the method of substitution, is a technique used in algebra to solve a system of linear equations. It involves manipulating the equations to eliminate one variable, allowing for the determination of the value of the remaining variable.
- Here's a step-by-step guide on how to use the elimination method to solve a system of linear equations with two variables:
- Write down the given system of equations. For example: Equation 1: $2x + 3y = 10$ Equation 2: $3x - 2y = 4$
- Choose one variable to eliminate. Look for a variable that has the same coefficient (positive and negative) in both equations. In this case, we can eliminate the variable "y" because it has the same coefficient but with opposite signs in both equations.
- Multiply one or both equations by appropriate constants so that the coefficients of the chosen variable are the same magnitude but opposite in sign. In this case, we can multiply Equation 1 by 2 and Equation 2 by 3 to make the coefficients of "y" the same magnitude: Equation 1 (multiplied by 2): $4x + 6y = 20$ Equation 2 (multiplied by 3): $9x - 6y = 12$
- Add or subtract the equations to eliminate the chosen variable. In this case, if we add the two equations together, the "y" term cancels out: $(4x + 6y) + (9x - 6y) = 20 + 12$ $13x = 32$
- Solve the resulting equation for the remaining variable. In this case, divide both sides of the equation by 13 to isolate "x": $13x = 32$ $x = 32/13$
- Substitute the value of the solved variable back into one of the original equations to find the value of the other variable. Let's use Equation 1: $2x + 3y = 10$ $2(32/13) + 3y = 10$ $64/13 + 3y = 10$ $3y = 10 - 64/13$ $3y = (130 - 64)/13$ $3y = 66/13$ $y = 66/13 * 1/3$ $y = 22/13$
- Check the solution by substituting the values of "x" and "y" into both original equations to ensure they satisfy both equations.
- So, the solution to the system of equations is $x = 32/13$ and $y = 22/13$.

- **The elimination method**, also known as the method of elimination or the method of substitution, is a technique used in algebra to solve a system of linear equations. It involves manipulating the equations to eliminate one variable, allowing for the determination of the value of the remaining variable.
- Here's a step-by-step guide on how to use the elimination method to solve a system of linear equations with two variables:
- Write down the given system of equations. For example: Equation 1: $2x + 3y = 10$ Equation 2: $3x - 2y = 4$
- Choose one variable to eliminate. Look for a variable that has the same coefficient (positive and negative) in both equations. In this case, we can eliminate the variable "y" because it has the same coefficient but with opposite signs in both equations.
- Multiply one or both equations by appropriate constants so that the coefficients of the chosen variable are the same magnitude but opposite in sign. In this case, we can multiply Equation 1 by 2 and Equation 2 by 3 to make the coefficients of "y" the same magnitude: Equation 1 (multiplied by 2): $4x + 6y = 20$ Equation 2 (multiplied by 3): $9x - 6y = 12$
- Add or subtract the equations to eliminate the chosen variable. In this case, if we add the two equations together, the "y" term cancels out: $(4x + 6y) + (9x - 6y) = 20 + 12$ $13x = 32$
- Solve the resulting equation for the remaining variable. In this case, divide both sides of the equation by 13 to isolate "x": $13x = 32$ $x = 32/13$
- Substitute the value of the solved variable back into one of the original equations to find the value of the other variable. Let's use Equation 1: $2x + 3y = 10$ $2(32/13) + 3y = 10$ $64/13 + 3y = 10$ $3y = 10 - 64/13$ $3y = (130 - 64)/13$ $3y = 66/13$ $y = 66/13 * 1/3$ $y = 22/13$
- Check the solution by substituting the values of "x" and "y" into both original equations to ensure they satisfy both equations.
- So, the solution to the system of equations is $x = 32/13$ and $y = 22/13$.

- **Quadratic equations** are second-degree polynomial equations that can be expressed in the form:

  $$ax^2 + bx + c = 0$$

  Here, 'x' represents the variable, and 'a', 'b', and 'c' are constants, where 'a' must be non-zero.

  To solve a quadratic equation, you can use the quadratic formula:

  $$x = (-b \pm \sqrt{(b^2 - 4ac)}) / (2a)$$

  The discriminant, represented as '$b^2 - 4ac$', determines the nature of the roots of the quadratic equation.

  If the discriminant is positive ($b^2 - 4ac > 0$), the equation has two distinct real roots.

  If the discriminant is zero ($b^2 - 4ac = 0$), the equation has one real root, also known as a repeated root or a perfect square.

  If the discriminant is negative ($b^2 - 4ac < 0$), the equation has two complex conjugate roots, which involve the imaginary unit 'i' (i.e., the square root of -1).

  By substituting the values of 'a', 'b', and 'c' into the quadratic formula, you can determine the solutions to the quadratic equation.

- **<u>The factorization method</u>** is one of the ways to solve a quadratic equation. A quadratic equation is an equation of the form ax^2 + bx + c = 0, where a, b, and c are constants and x is the variable. To solve a quadratic equation using factorization, we aim to express the equation as a product of two binomial expressions and set each factor equal to zero.
- Here's an example to illustrate the factorization method:
- Let's solve the quadratic equation: x^2 - 5x + 6 = 0
- Step 1: Write down the equation in the form ax^2 + bx + c = 0. In our example, a = 1, b = -5, and c = 6. So the equation becomes x^2 - 5x + 6 = 0.
- Step 2: Factorize the quadratic expression on the left-hand side. We need to find two binomial expressions whose product is equal to x^2 - 5x + 6. In this case, the factorization is (x - 2)(x - 3). Therefore, we can rewrite the equation as follows: (x - 2)(x - 3) = 0.
- Step 3: Set each factor equal to zero and solve for x. (x - 2) = 0 --> x = 2 (x - 3) = 0 --> x = 3
- Step 4: Check the solutions. Plug the values of x back into the original equation to verify if they satisfy the equation. In this case, substituting x = 2 and x = 3 into x^2 - 5x + 6 = 0 yields: For x = 2: 2^2 - 5(2) + 6 = 4 - 10 + 6 = 0 (satisfied) For x = 3: 3^2 - 5(3) + 6 = 9 - 15 + 6 = 0 (satisfied)
- Therefore, the solutions to the quadratic equation x^2 - 5x + 6 = 0 are x = 2 and x = 3.

Formula Method

- The quadratic equation is a second-degree polynomial equation in a single variable, and it has the general form: $ax^2 + bx + c = 0$. To solve this equation using the quadratic formula, you can follow these steps:
- **Step 1:** Identify the values of a, b, and c in the quadratic equation.
- **Step 2:** Substitute the values of a, b, and c into the quadratic formula: $x = (-b \pm \sqrt{b^2 - 4ac}) / (2a)$
- **Step 3:** Simplify the expression under the square root (√).
- **Step 4:** Compute the values of x using the quadratic formula.
- Let's go through an example to illustrate the process:
- **Example: Solve the quadratic equation $2x^2 - 5x + 3 = 0$ using the quadratic formula.**
- **Step 1:** Identify the values of a, b, and c. In this equation, $a = 2$, $b = -5$, and $c = 3$.
- **Step 2:** Substitute the values into the quadratic formula. $x = (-(-5) \pm \sqrt{(-5)^2 - 4 * 2 * 3}) / (2 * 2) = (5 \pm \sqrt{25 - 24}) / 4 = (5 \pm \sqrt{1}) / 4$
- **Step 3:** Simplify the expression under the square root. $\sqrt{1} = 1$
- **Step 4:** Compute the values of x. $x1 = (5 + 1) / 4 = 6 / 4 = 1.5$
- $x2 = (5 - 1) / 4 = 4 / 4 = 1$
- Therefore, the solutions to the quadratic equation $2x^2 - 5x + 3 = 0$ are $x = 1.5$ and $x = 1$.

Thank You

By Swetha M M